CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

   (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

   (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

   (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

   (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

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a) $G'(t) = 45\left(\frac{2t}{18}\right)\left(-\sin\left(\frac{t^2}{18}\right)\right)$

$= -5t\sin\left(\frac{t^2}{18}\right)$

$G'(5) = -25\sin\left(\frac{25}{18}\right)$

≈ -24.588 The rate of gravel arriving at the plant is decreasing at a rate of 24.588 tons per hour per hour at time $t = 5$ hours.

b) Total amount arriving

$= \int_0^8 \left(90 + 45\cos\left(\frac{t^2}{18}\right)\right)dt$

≈ 825.551 tons of gravel

c) Plant processes gravel at $\frac{100 \text{ tons}}{\text{hr.}}$

Compare rates of unprocessed and processed at $t = 5$:

$G(5) - 100$

$90 + 45\cos\left(\frac{25}{18}\right) - 100$

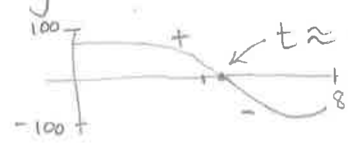
≈ -1.859 tons/hr.

So the amount of unprocessed gravel is decreasing by 1.859 tons/hr.

d) Find when $G(t) - 100$ goes from positive to negative:

graph:

$y = 90 + 45\cos\left(\frac{t^2}{18}\right) - 100$



Max amount of unprocessed gravel:

$500 + \int_0^{\approx 4.923}\left[90 + 45\cos\left(\frac{t^2}{18}\right)\right]dt - 100(\approx 4.923)$

$\boxed{\approx 635.376 \text{ tons}}$

2. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

    (a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2. Speed $= |v(t)|$

    (b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

    (c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

    (d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

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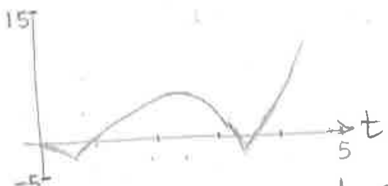
a) $2 = \left|-2 + (t^2 + 3t)^{6/5} - t^3\right|$

$0 = \left|-2 + (t^2 + 3t)^{6/5} - t^3\right| - 2$

graph & find zeros on $\underline{[2,4]}$



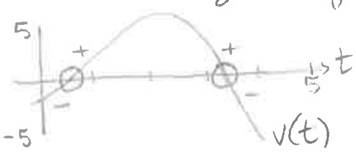
$t \approx 3.128$ and 3.473

b) $s(t) = 10 + \int_0^t \left[-2 + (x^2 + 3x)^{6/5} - x^3\right] dx$

$s(5) = 10 + \int_0^5 \left[-2 + (t^2 + 3t)^{6/5} - t^3\right] dt$

≈ -9.207

c) Particle will change direction where $v(t)$ changes sign.



$t \approx 0.536$ & 3.318

d) $v(4) \approx -11.476$

$v'(4) \approx -22.296$

Velocity is negative and decreasing at $x = 4$, so speed is increasing.

(Speed is increasing when velocity and acceleration have the same sign.)

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure. use Avg rate of change btwn. 3 & 4 min.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer. MVT → p. 164

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem. average value

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

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a) $C'(3.5) \approx \frac{C(4) - C(3)}{4 - 3}$

$\approx 12.8 - 11.2$

$C'(3.5) \approx 1.6$ ounces of coffee per minute.

b) Mean Value Th:

on $[2,4]$ $C'(t) = \frac{12.8 - 8.8}{4-2}$

$C'(t) = 2$ ✓ $\boxed{\text{yes}}$

Since C is differentiable, it is continuous on $[2,4]$

there is at least one time, t, on $(2,4)$ where

$C'(t) = 2$

c) $n = 3$, $[0,6]$, $\Delta x = 2$, midpts: 1, 3, 5

$\frac{1}{6}\int_0^6 C(t)\,dt \approx \frac{2}{6}\left[C(1) + C(3) + C(5)\right]$

$\approx \frac{1}{3}(5.3 + 11.2 + 13.8)$

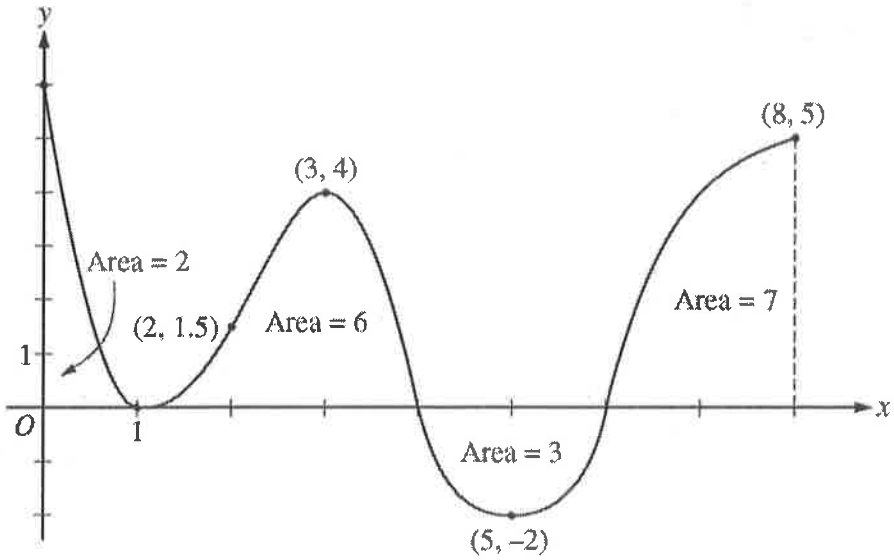
≈ 10.1 ounces is the average amount of coffee in the cup over the first 6 minutes.

d) $B'(t) = 6.4e^{-0.4t}$

$B'(5) = 6.4e^{-0.4(5)}$

$= \frac{6.4}{e^2}$

The amount of coffee in the cup is increasing at a rate of $\frac{6.4}{e^2}$ oz/min.



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

$f(0)$?

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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a) f has local minimum where f' goes from negative to positive

$x = 6$

b) Compare outcomes, $f(x)$, at endpts and local min.

$f(x) = \int f'(x)\,dx$

$\boxed{f(8) = 4}$

$f(6) = f(8) - \int_6^8 f'(x)\,dx$

$= 4 - 7$

$\boxed{f(6) = -3}$

$f(0) = f(6) - \int_0^6 f'(x)\,dx$

$= -3 - (2 + 6 - 3) = \boxed{-8}$

$\boxed{\text{Absolute Minimum} = -8}$

c) f conc. down where f' is decreasing: $(0,1) \cup (3,5)$

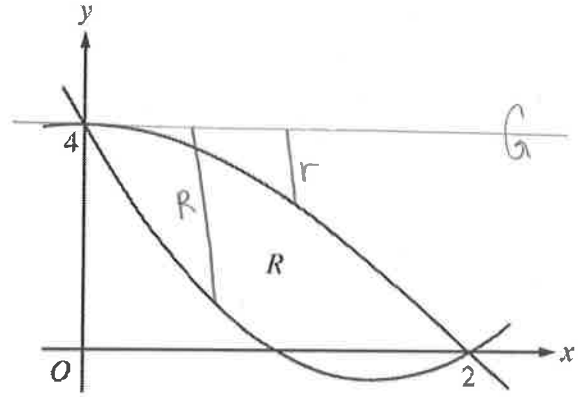
f increasing where f' is positive: $(0,1) \cup (1,4) \cup (6,8)$

f is both concave down and increasing on $\boxed{(0,1) \cup (3,4)}$

d) $g'(x) = 3(f(x))^2 \cdot f'(x)$

$g'(3) = 3(-\frac{5}{2})^2 \cdot 4$

$\boxed{g'(3) = 75}$ Slope of tangent @ $x = 3$



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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a) $A = \int_0^2 \left[4\cos\left(\frac{1}{4}\pi x\right) - (2x^2 - 6x + 4)\right] dx$

$A = 4 \cdot \frac{4}{\pi} \int_0^2 \frac{\pi}{4} \cos\left(\frac{\pi}{4}x\right) dx - 2\int_0^2 (x^2 - 3x + 2) dx$

$= \frac{16}{\pi}\left[\sin \frac{\pi x}{4}\right]_0^2 - 2\left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^2$

$= \frac{16}{\pi}\left(\sin \frac{\pi}{2} - \sin 0\right) - 2\left(\frac{8}{3} - 6 + 4 - 0\right)$

$= \frac{16}{\pi} - \frac{16}{3} + 12 - 8$

$= \frac{16}{\pi} - \frac{4}{3}$

b) $R = 4 - (2x^2 - 6x + 4)$

$R = -2x^2 + 6x$

$r = 4 - 4\cos\left(\frac{\pi}{4}x\right)$

$V = \pi \int_0^2 \left[(-2x^2 + 6x)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$

c) side $= 4\cos\left(\frac{\pi}{4}x\right) - (2x^2 - 6x + 4)$

$V = \int_0^2 \left(4\cos\left(\frac{\pi}{4}x\right) - 2x^2 + 6x - 4\right)^2 dx$

6. Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

   (a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

   (b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

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a) slope $= e^0(3(1)^2 - 6(1))$

$= 1(-3)$

$y = -3(x-1)$

$\boxed{y = -3x + 3}$

$f(1.2) \approx -3(1.2) + 3$

$\boxed{f(1.2) \approx -0.6}$

b) $\int \frac{1}{e^y}\,dy = \int (3x^2 - 6x)\,dx$

$-e^{-y} = x^3 - 3x^2 + C$

$-e^0 = 1 - 3 + C$

$C = 1$

$-e^{-y} = x^3 - 3x^2 + 1$

$e^{-y} = -x^3 + 3x^2 - 1$

$-y = \ln|-x^3 + 3x^2 - 1|$

$\boxed{y = -\ln|-x^3 + 3x^2 - 1|}$